PolyCLEAN: A Polyatomic Frank-Wolfe Algorithm for Interferometric Imaging

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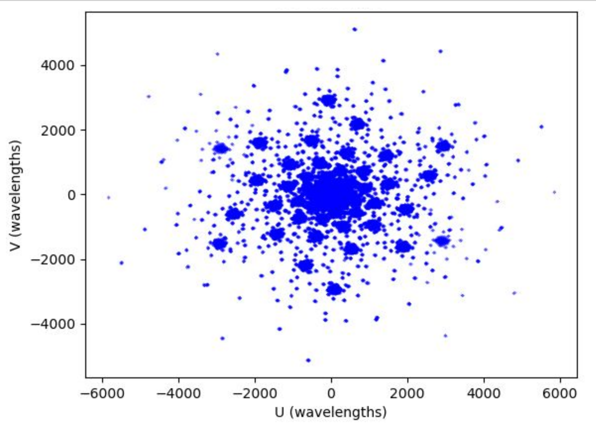
Interferometric Imaging Measurement Equation

Interferometric imaging :

- Ill-posed inverse problem
- Noisy measurements
- Wide field of view
  = large volumes of data



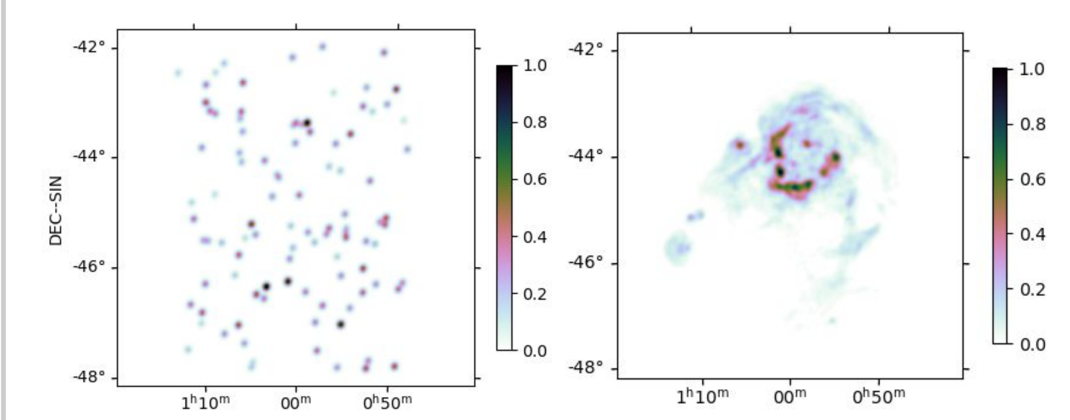
UV coverage

Visibility measurements — Sky Image

$$\boxed{\mathbf{V} = \mathbf{\Phi}\mathbf{I}}$$

$$V_k = (\mathbf{\Phi}\mathbf{I})_k = \sum_{i,j} \frac{e^{-j2\pi(u_k\ell_i + v_k m_j)}}{\sqrt{1-\ell_i^2 - m_j^2}} \mathcal{W}(\ell_i, m_j; w_k) I_{i,j}$$

Baselines : $\mathbf{b}_k = (u_k, v_k, w_k) \in \mathbb{R}^3$



Point sources — Extended sources

Two main approaches based on sparsity priors

1. Sparse Decomposition with CLEAN

CLEAN assumption:

$$\mathbf{I}_{\mathrm{CLEAN}} = \sum_n a_n \delta_{s_n}$$

Algorithm 1 Högbom CLEAN Algorithm (Major cycles only)

Initialisation : $\mathbf{I}^{(0)} = \mathbf{0}$, $\mathbf{I}_D = \mathbf{\Phi}^*\mathbf{V}$, $\alpha > 0$

for $k = 1, 2, \cdots, k_{\max}$ **do**
1. Compute the dirty residual: $\mathbf{I}_R^{(k)} = \mathbf{I}_D - \mathbf{\Phi}^*\mathbf{\Phi}\mathbf{I}^{(k-1)}$
2. Find the next reconstructed source: $s^{(k)} = \arg\max\ \mathbf{I}_R^{(k)}$
3. Update the iterate: $\mathbf{I}^{(k)} = \mathbf{I}^{(k-1)} + \alpha\delta_{s^{(k)}}$

end for

- ✅ Efficient implementation (fast algorithm, convolutions)
- ✅ Calibration-compliant
- ✅ Scalable (sparse iterates)
- ❌ Objective function unclear
- ❌ Stopping criterion unclear
- ❌ Dictionary-based priors

2. Bayesian MAP Estimation

$$\arg\min \underbrace{\frac{1}{2}\|\mathbf{V} - \mathbf{\Phi}\mathbf{I}\|_2^2}_{\text{Likelihood}} + \underbrace{h(\mathbf{I})}_{\text{Prior}}$$

Convex optimization problem, solved with **proximal solvers**

Classical example: **LASSO problem**

$$h(\mathbf{I}) = \lambda \|\mathbf{I}\|_1, \quad \lambda > 0$$

Proximal Gradient Descent :

$$\mathbf{I}^{(k+1)} \leftarrow \mathrm{Soft}_{\lambda\tau}\left(\mathbf{I}^{(k)} - \tau\nabla f(\mathbf{I}^{(k)})\right)$$

- ✅ Flexible and interpretable priors
- ✅ Principled fast solvers
- ❌ Require CLEAN-based calibration
- ❌ Dense iterates - difficult to scale

A Unifying Algorithm: PolyCLEAN

We propose **PolyCLEAN**:
a principled algorithm to solve a **LASSO problem**
while maintaining a **CLEAN-like** atomic architecture

$$\arg\min_{\mathbf{I}\geqslant 0} \frac{1}{2}\|\mathbf{V} - \mathbf{\Phi}\mathbf{I}\|_2^2 + \lambda\|\mathbf{I}\|_1$$

Sparse iterates : scalable method
Explicit prior : principled algorithm and stopping criterion

Generalization to extended sources reconstruction with sparse representations over wavelet coefficients:

$$\arg\min_{\mathbf{\Theta}} \frac{1}{2}\|\mathbf{V} - \mathbf{\Phi}\underbrace{\mathbf{\Psi}\mathbf{\Theta}}_{\text{Image } \mathbf{I}}\|_2^2 + \lambda\underbrace{\|\mathbf{\Theta}\|_1}_{\text{Coefficients}}$$

Algorithm 2 PolyCLEAN

Initialisation : $\mathbf{I}^{(0)} = \mathbf{0}$, $\mathcal{S}^{(0)} = \mathrm{Supp}(\mathbf{I}^{(0)}) = \emptyset$, $\mathbf{I}_D = \mathbf{\Phi}^*\mathbf{V}$

while `stopping_criterion`$(\mathbf{I}^{(k)})$ not reached **do**
1. Compute the dirty residual: $\mathbf{I}_R^{(k)} = \mathbf{I}_D - \mathbf{\Phi}^*\mathbf{\Phi}\mathbf{I}^{(k-1)}$
2. Place many candidate sources: $s_1^{(k)}, s_2^{(k)}, \cdots =$ `highest_level_set` $(\mathbf{I}_R^{(k)})$
   Update active set : $\mathcal{S}^{(k)} \leftarrow \mathcal{S}^{(k-1)} \cup \{s_1^{(k)}, s_2^{(k)}, \dots\}$
3. Update the iterate:

$$\mathbf{I}^{(k)} = \arg\min_{\substack{\mathrm{Supp}(\mathbf{I})\subset\mathcal{S}^{(k)}\\ \mathbf{I}\geqslant 0}} \frac{1}{2}\|\mathbf{V} - \mathbf{\Phi}\mathbf{I}\|_2^2 + \lambda\|\mathbf{I}\|_1 \qquad \text{(R)}$$

end while

Polyatomic Frank-Wolfe Algorithm

- Place many sources at each iteration
  - Numerous candidates at start
  - Less candidates later on
- Solved with a truncated ISTA
  - Restricted support : reduced dimensions of the problem for a scalable step
  - Early stop of the solver, with increasing accuracy along PolyCLEAN iterations

Ref: Jarret et al., 2021

Efficient computation of the forward operator $\mathbf{\Phi}$ with NUFFT
Ref: M. Simeoni et al.

Numerical Experiments

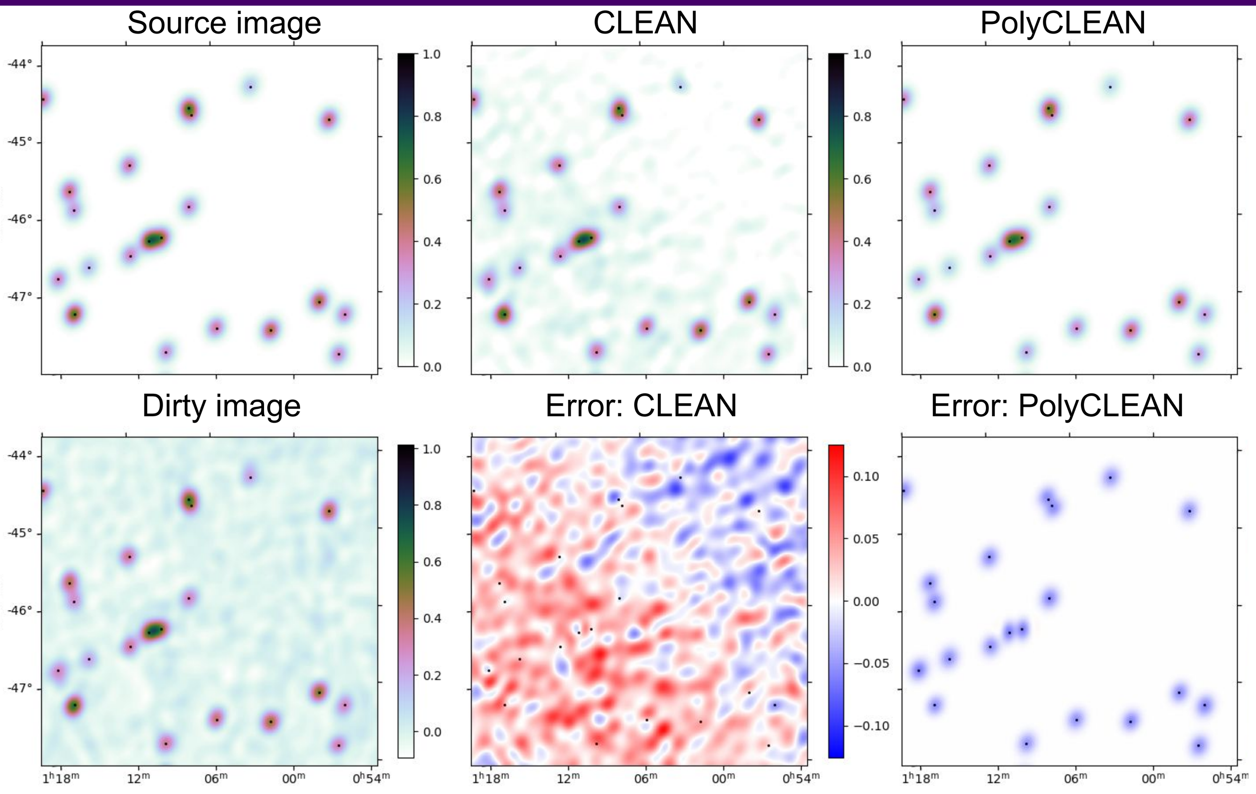
Simulated Image Parameters

Image size:	1024*1024
Radius of the stations:	rmax
FOV (deg):	16
Number of sources:	100

Post processing: PolyCLEAN+

Account for the shrinking of the LASSO with least squares reweighting

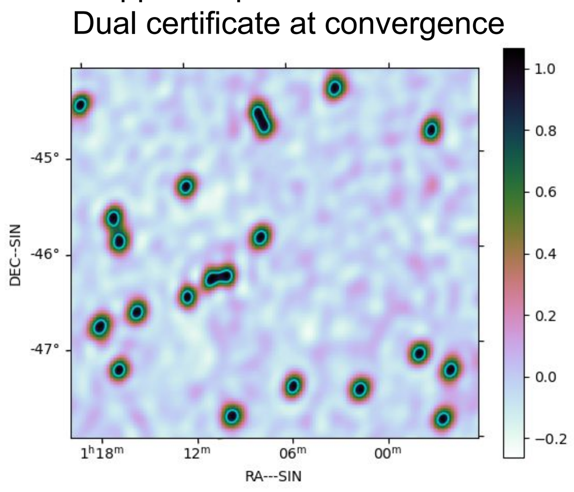
$$\arg\min_{\substack{\mathbf{I}\geqslant 0\\ \mathrm{Supp}(\mathbf{I})\subset\mathrm{Supp}(\mathbf{I}^*)}} \|\mathbf{V} - \mathbf{\Phi}\mathbf{I}\|_2^2$$



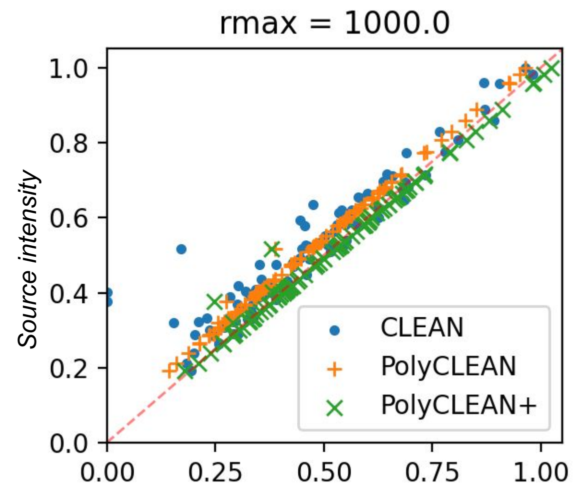
Metrics

	rmax = 500 (~ 18500 baselines)				rmax = 1000 (~ 31500 baselines)			
	CLEAN	APGD	Poly CLEAN	Poly CLEAN+	CLEAN	APGD	Poly CLEAN	Poly CLEAN+
Duration (s)	237.0	246.1	**38.7**	**37.9**	251.1	247.9	**125.3**	**72.2**
MSE	7.4e-4	**2.3e-5**	**2.3e-5**	**9.4e-7**	4.0e-3	**1.6e-5**	**1.7e-5**	**1.1e-6**
Sparsity	9587	1644	1398	1218	9700	1218	539	785

Support of potential solutions:
Dual certificate at convergence



rmax = 1000.0



Intensity matching on 9x9 superpixels

Take-Home Messages

PolyCLEAN brings the robustness and interpretability of **optimization** to radio interferometric imaging in a fast and efficient manner, with an atomic **CLEAN-like** method.

	CLEAN	MAP Estimation	PolyCLEAN
Sparse iterates	✅	❌	✅
Flexible priors	❌	✅	~
Fast solvers	~	✅	✅
Calibration compliant	✅	❌	✅
Objective function	❌	✅	✅

PolyCLEAN also handles extended celestial sources using sparse wavelet coefficients representation.

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