Interferometric Imaging in Radio Astronomy with the Sparsity-Promoting Frank-Wolfe Algorithm

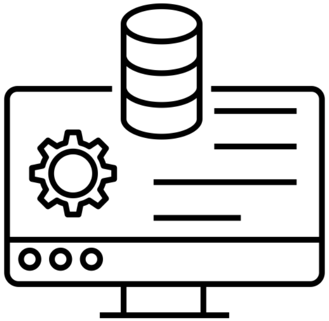
Adrian Jarret, PhD student @EPFL/LCAV

joint work with Matthieu Simeoni, Julien Fageot, Martin Vetterli

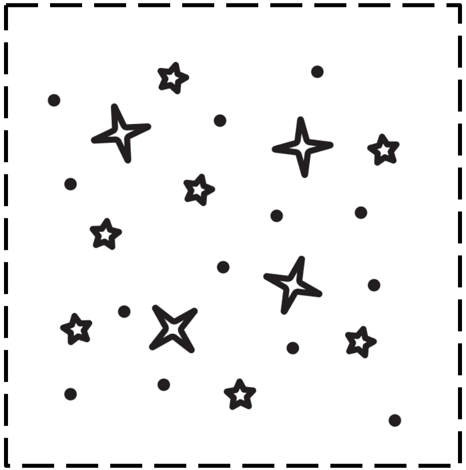
SKA Days 2021

INTRODUCTION

Measurement



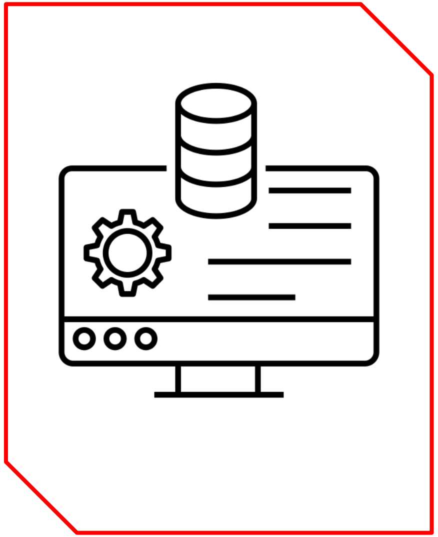
Data Processing



Sky Image

INTRODUCTION

Measurement



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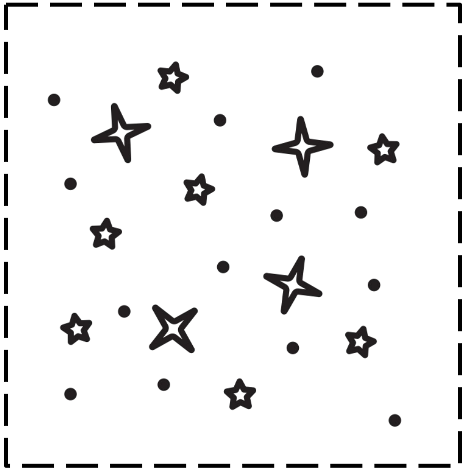


Sky Image

Data Model and Assumptions

From interferometric measurements to Inverse Problem

- Fourier-type measurements[1] :

Van Cittert-Zernike theorem

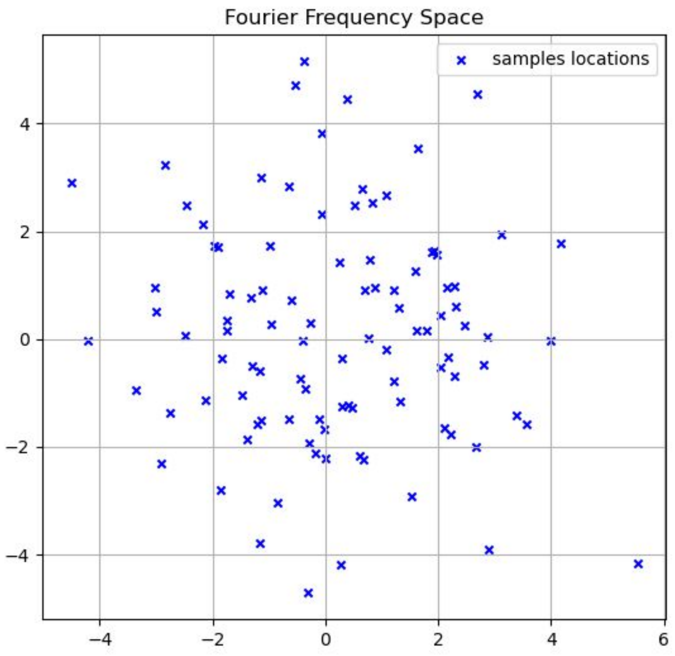
$$\mathcal{V}(u,v) = \mathcal{F}\{I\}(u,v) = \iint I(l,m)e^{-2i\pi(ul+vm)}\mathrm{d}l\mathrm{d}m$$

- Linear inverse problem:

$$\mathbf{V} = \mathbf{\Phi}(I) \quad \in \mathbb{C}^L$$

Visibility

Sky Image



Data Model and Assumptions

Optimization Problem

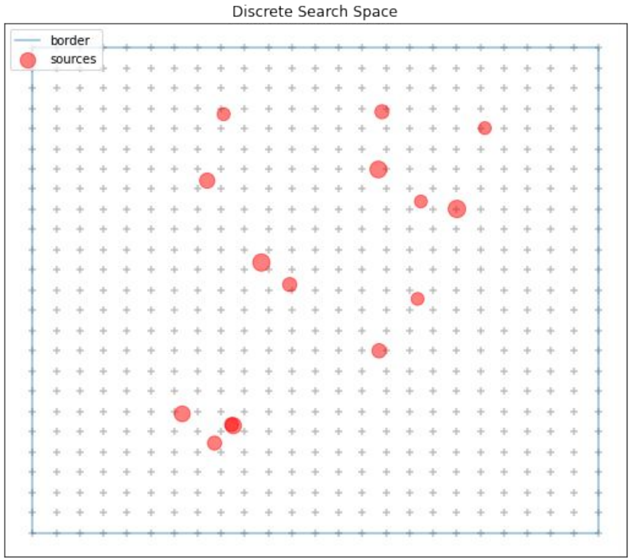
- Discretization:

$$I \quad \approx \quad \boldsymbol{\beta} \in \mathbb{R}^N \quad \Rightarrow \quad \mathbf{V} = \mathbf{G}\boldsymbol{\beta}$$

- Our strategy, LASSO[2]:

$$\boxed{\text{Minimize :} \quad \frac{1}{2}\left\|\mathbf{V} - \mathbf{G}\boldsymbol{\beta}\right\|_2^2 + \lambda \left\|\boldsymbol{\beta}\right\|_1}$$

- Classical solvers: PDS[3], APGD[4], FISTA[5]



Numerical Strategy

Our contribution: Reweighted Frank-Wolfe

Algorithm 1 Reweighted Frank-Wolfe (RFW)

Candidate locations: $S_k \leftarrow \emptyset$

for $k = 1, \cdots, k_{\max}$ **do**

1. Estimate new location(s): $i_k \in \underset{i \in \{1,\dots,N\}}{\arg\max} \left| \mathbf{G}^* \left(\mathbf{V} - \mathbf{G}\boldsymbol{\beta}_k \right) \right|_i$

1.(bis) Update locations: $S_{k+1} \leftarrow S_k \cup \{i_k\}$

2. Complete best reweighting: $\boldsymbol{\beta}_{k+1} \leftarrow \underset{\mathrm{Supp}(\boldsymbol{\beta}) \subset S_{k+1}}{\arg\min} \frac{1}{2} \left\| \mathbf{V} - \mathbf{G}\boldsymbol{\beta} \right\|_2^2 + \lambda \left\| \boldsymbol{\beta} \right\|_1$

end for

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Residual Image

CLEAN-like

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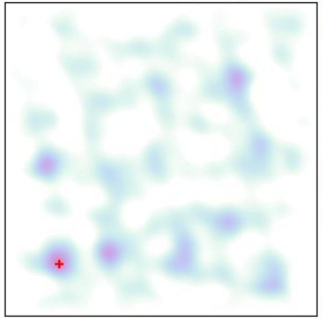
end for

Residual Dirty Image



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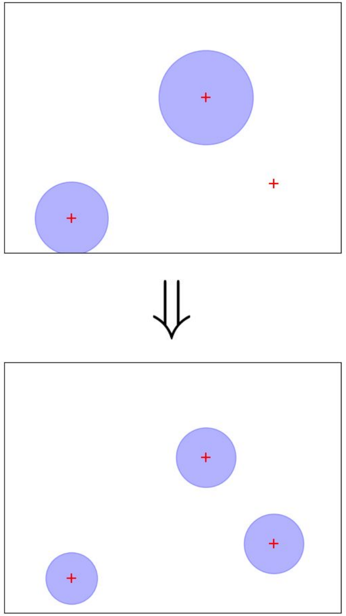
end for

Restricted
Support LASSO



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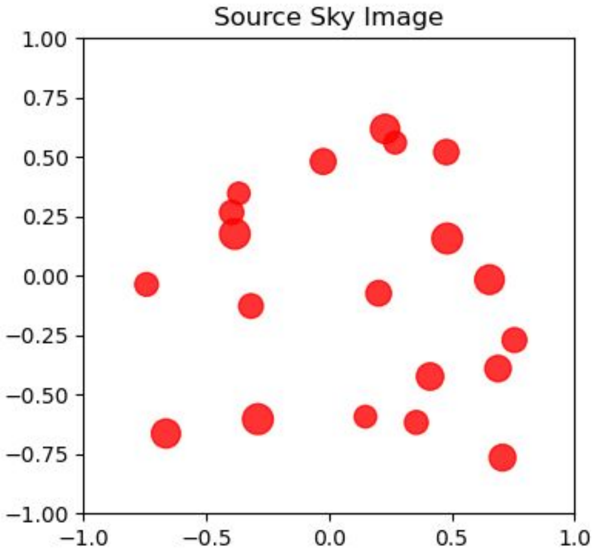
1.(bis) Update locations: $S_{k+1} \leftarrow S_k \cup \{i_k\}$

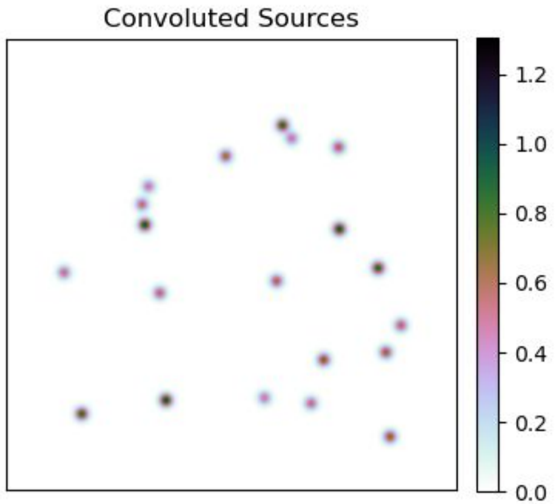
2. Complete best reweighting: $\boldsymbol{\beta}_{k+1} \leftarrow \arg\min_{\mathrm{Supp}(\boldsymbol{\beta}) \subset S_{k+1}} \frac{1}{2}\left\|\mathbf{V} - \mathbf{G}\boldsymbol{\beta}\right\|_2^2 + \lambda \left\|\boldsymbol{\beta}\right\|_1$

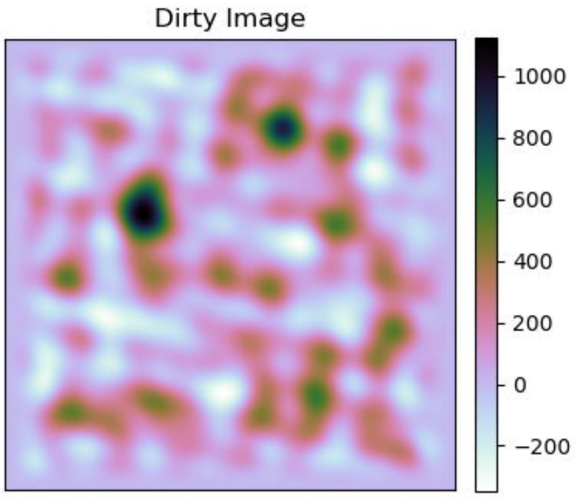
end for

Performances on Simulated Data

Data Simulation

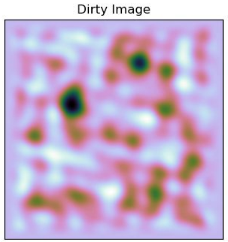


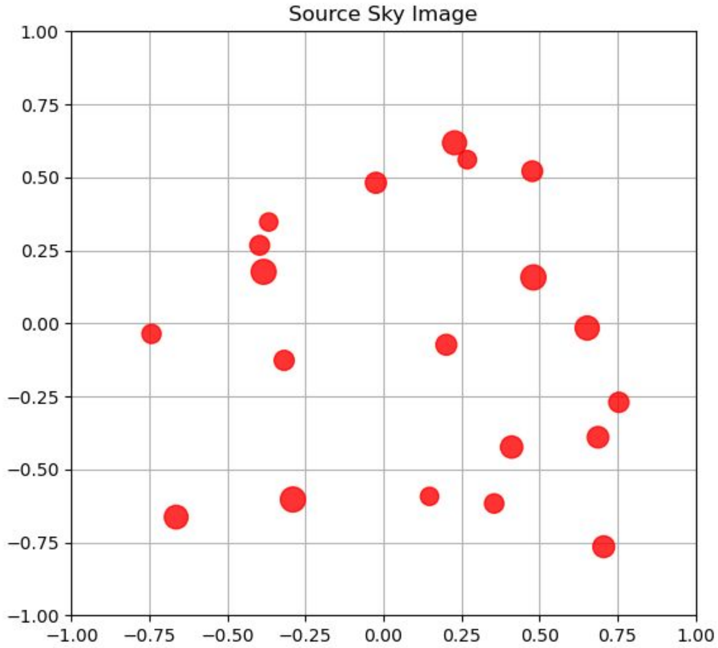


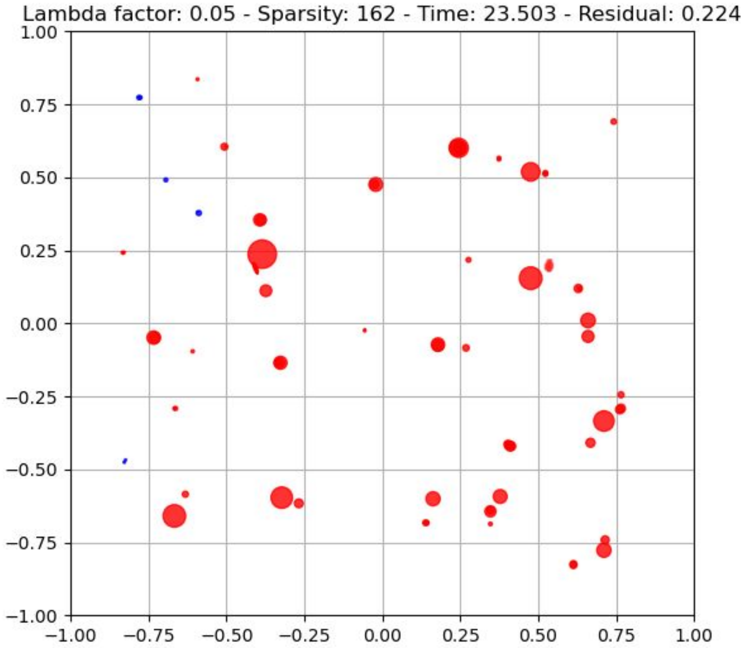


Performances on Simulated Data

Reweighted FW

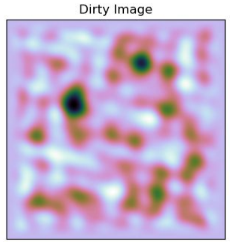


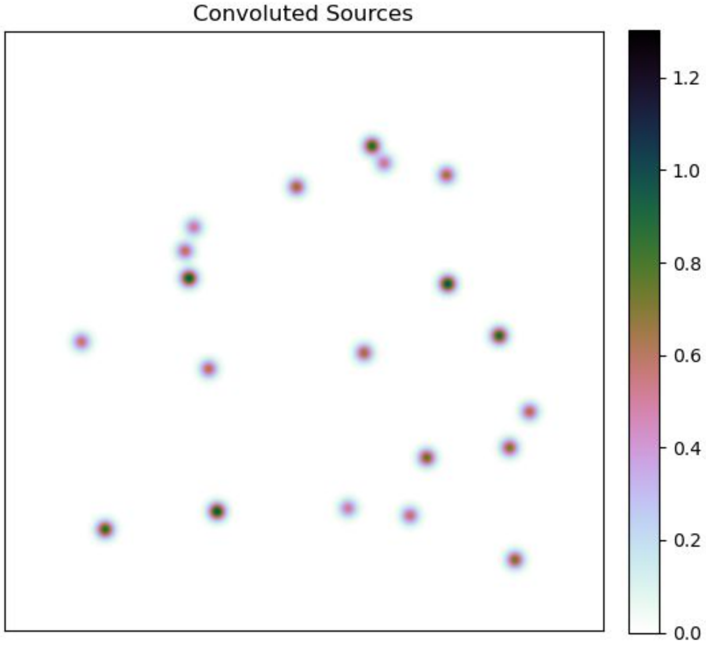


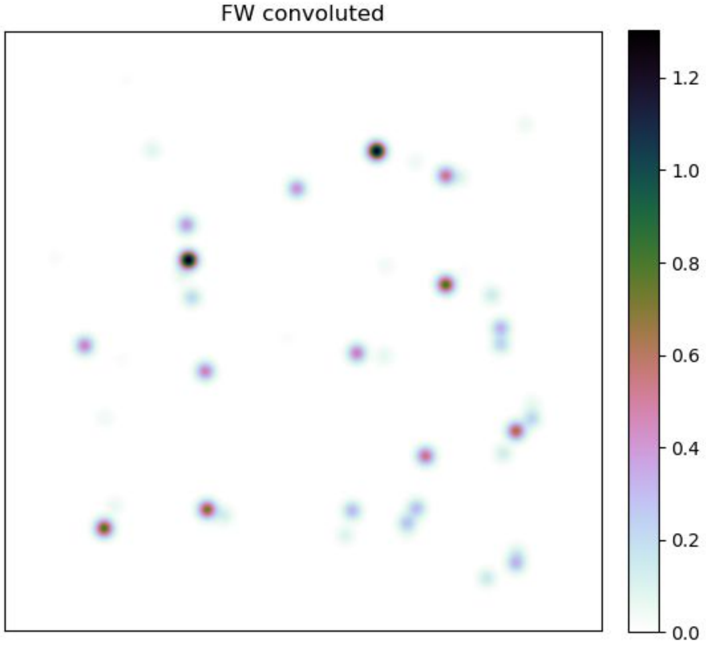


Performances on Simulated Data

Reweighted FW



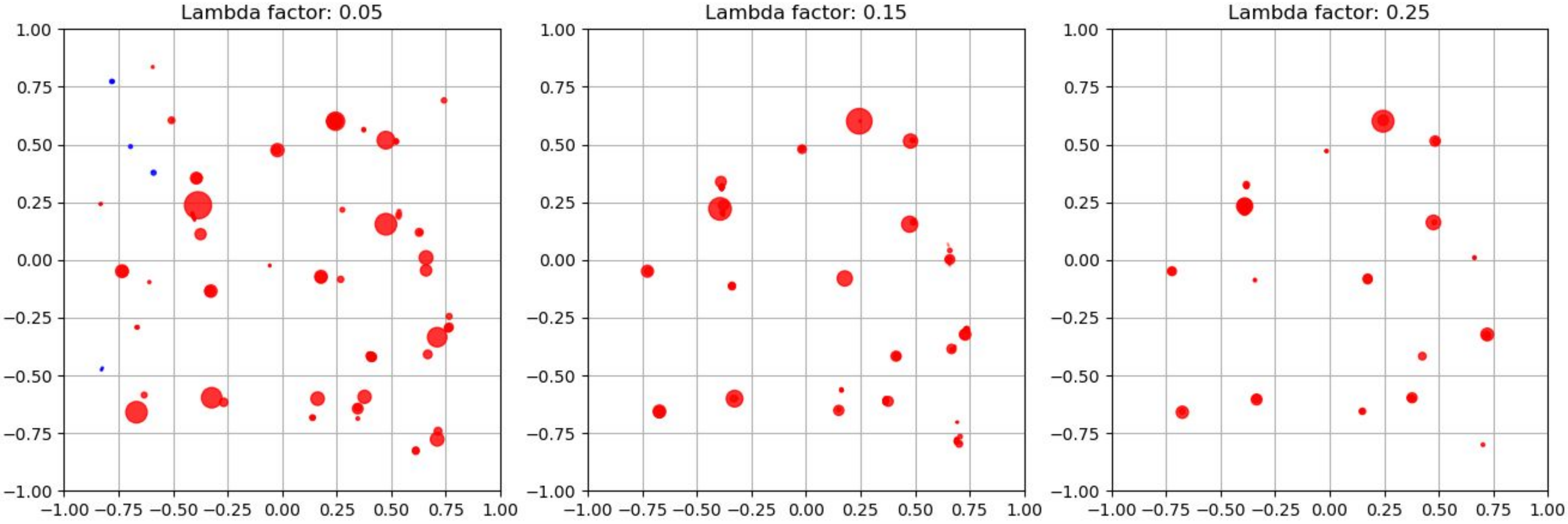




Performances on Simulated Data

Effect of λ

$$\text{Minimize}: \quad \frac{1}{2}\left\|\mathbf{V} - \mathbf{G}\boldsymbol{\beta}\right\|_2^2 + \lambda\left\|\boldsymbol{\beta}\right\|_1$$



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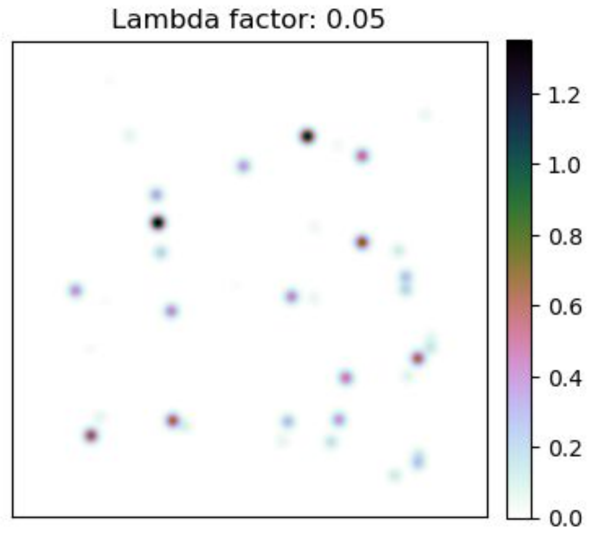
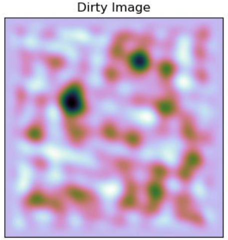
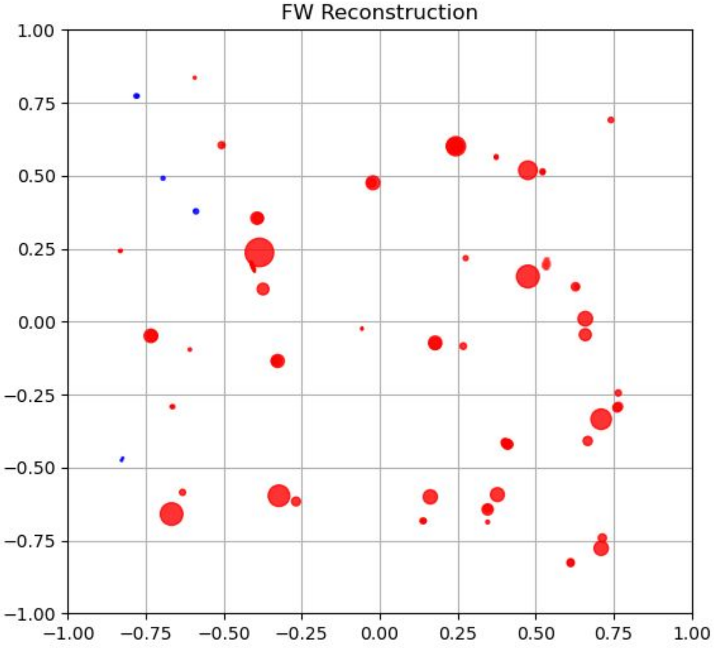


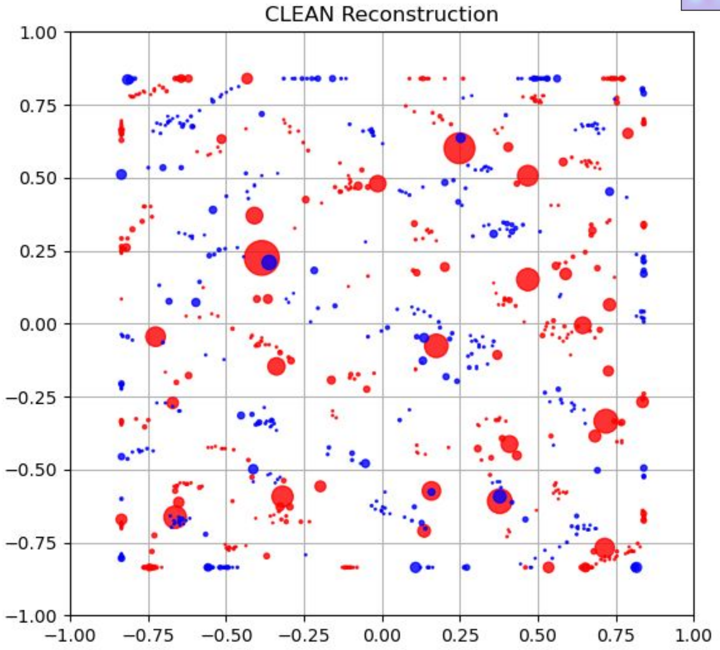
Performances on Simulated Data

Comparison with CLEAN





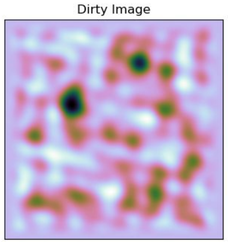
Time: 22.949 - Residual: 0.224

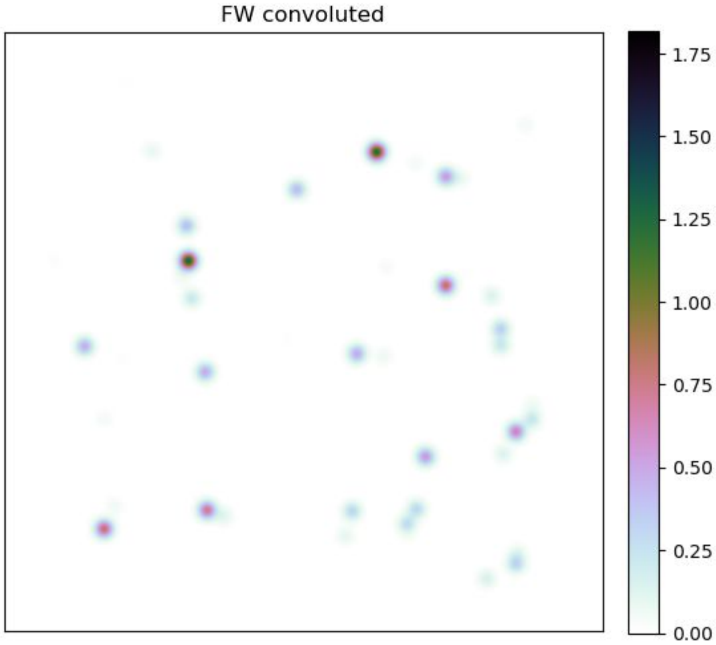


Time: 42.482 - Residual: 0.115

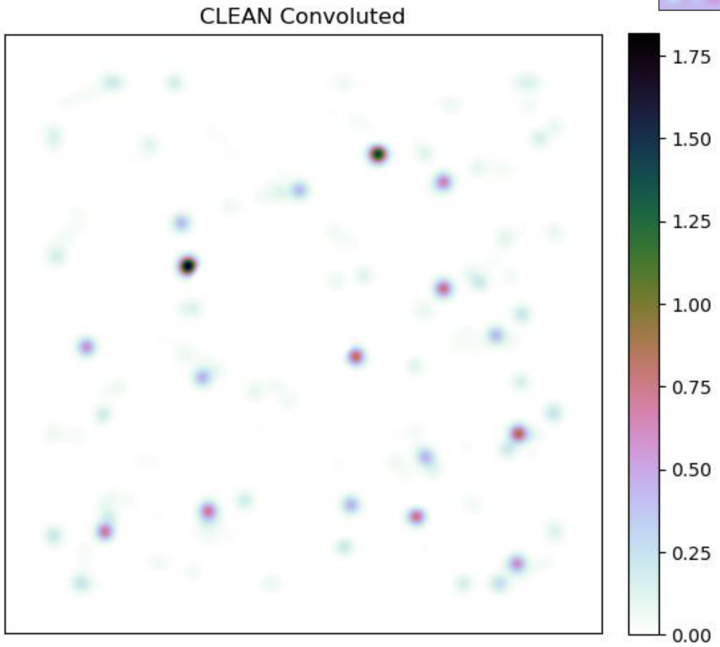
Performances on Simulated Data

Comparison with CLEAN





RRMSE: 0.746



RRMSE: 0.985

Conclusion

Is Reweighted Frank-Wolfe a decent contender for CLEAN?

- Competitive running time
- Improved reconstruction accuracy
- Natural stopping criterion

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Is Reweighted Frank-Wolfe a decent contender for CLEAN?

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- Well suited for RA
  - Greedy = adapted to sparse problems
  - Parametric reconstruction (penalization parameter) = adjustable

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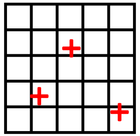
Is Reweighted Frank-Wolfe a decent contender for CLEAN?

- Competitive running time
- Improved reconstruction accuracy
- Natural stopping criterion

- Well suited for RA
  - Greedy = adapted to sparse problems
  - Parametric reconstruction (penalization parameter) = adjustable

- Positivity constraint
- Natural extension to continuous data
  - Beyond-the-grid precision
  - Principled theoretical framework



References

[1] Veen, Alle-Jan van der, Stefan J. Wijnholds, and Ahmad Mouri Sardarabadi. 2019. “Signal Processing for Radio Astronomy.” In *Handbook of Signal Processing Systems.*

[2] Tibshirani, Robert. 1996. “Regression Shrinkage and Selection via the Lasso.” *Journal of the Royal Statistical Society. Series B (Methodological)* 58 (1): 267–88.

[3] Condat, Laurent. 2013. “A Primal–Dual Splitting Method for Convex Optimization Involving Lipschitzian, Proximable and Linear Composite Terms.” *Journal of Optimization Theory and Applications* 158 (2): 460–79.

[4] Liang, Jingwei, Tao Luo, and Carola-Bibiane Schönlieb. 2021. “Improving ‘Fast Iterative Shrinkage-Thresholding Algorithm’: Faster, Smarter and Greedier.” *ArXiv:1811.01430 [Math]*, January.

[5] Beck, Amir, and Marc Teboulle. 2009. “A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems.” *SIAM Journal on Imaging Sciences* 2 (1): 183–202.

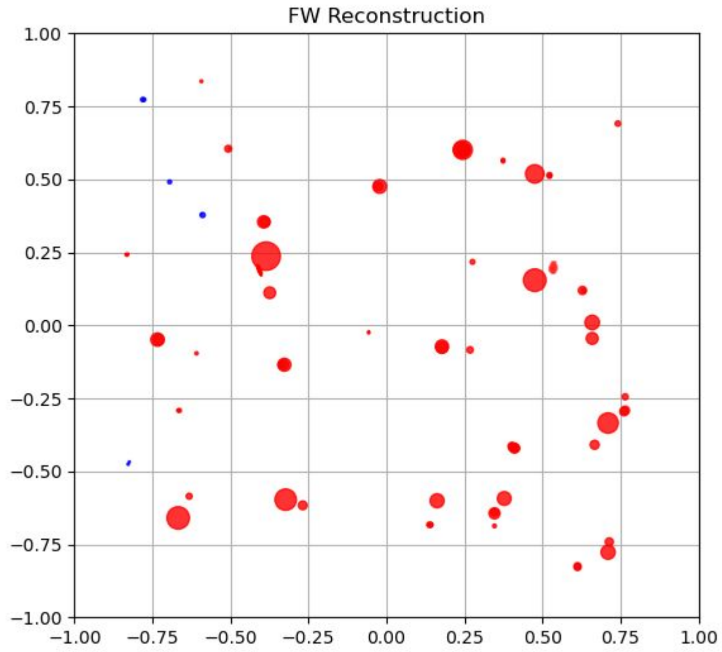
[6] Frank, Marguerite, and Philip Wolfe. 1956. “An Algorithm for Quadratic Programming.” *Naval Research Logistics Quarterly* 3 (1–2): 95–110.

Appendice

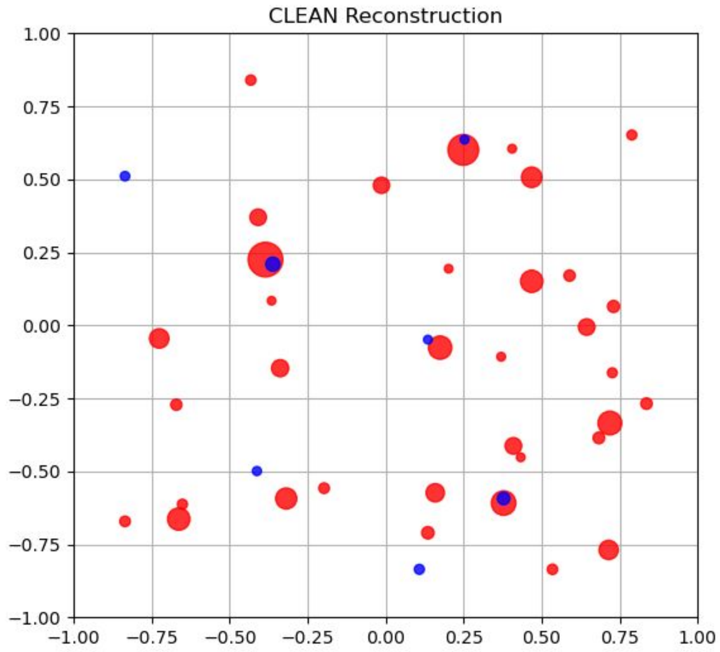
If there is enough time

Performances on Simulated Data

Improved CLEAN



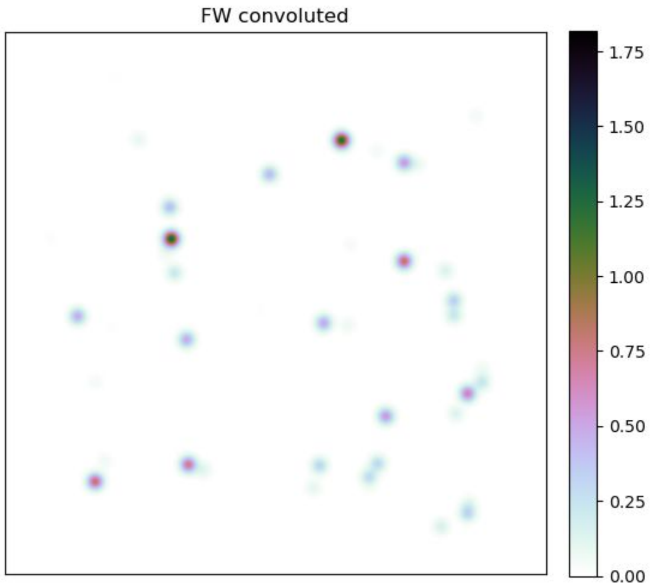
Time: 22.949 - Residual: 0.224



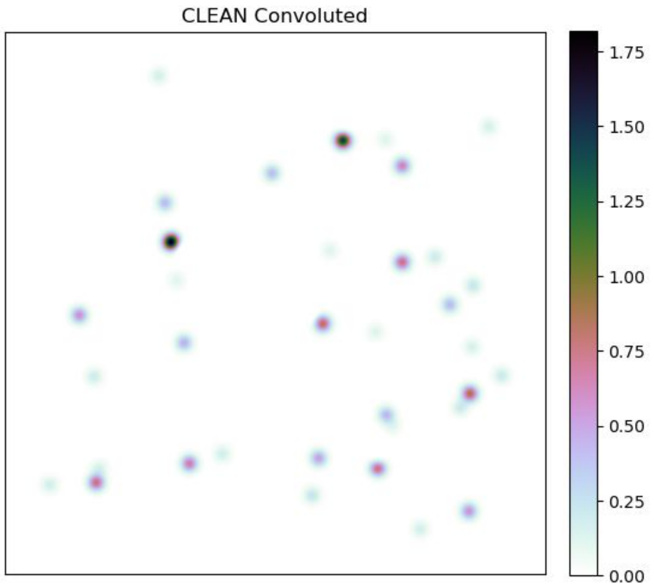
Time: 3.017 - Residual: 0.197

Performances on Simulated Data

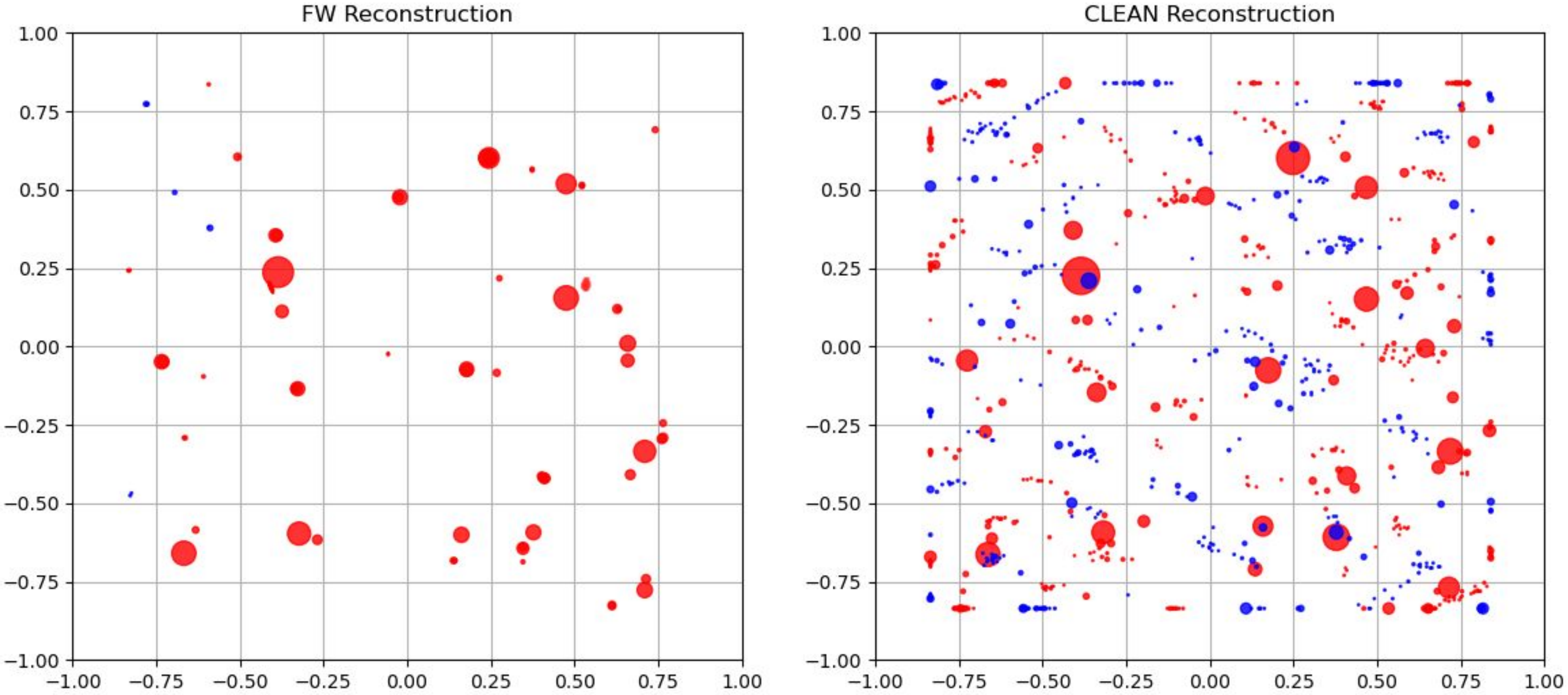
Improved CLEAN



RRMSE: 0.746



RRMSE: 0.936


FW Reconstruction
CLEAN Reconstruction
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00